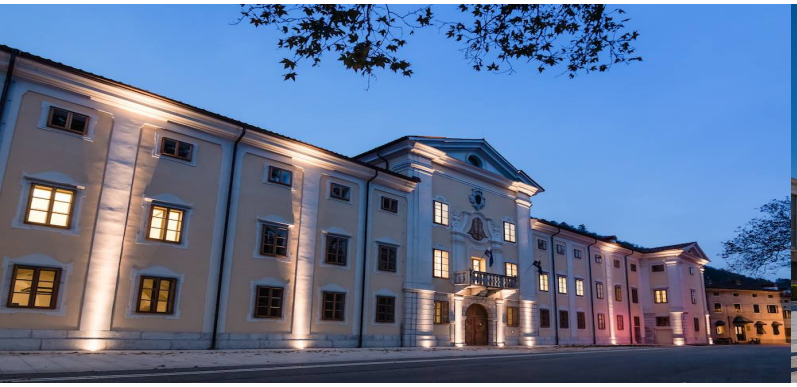
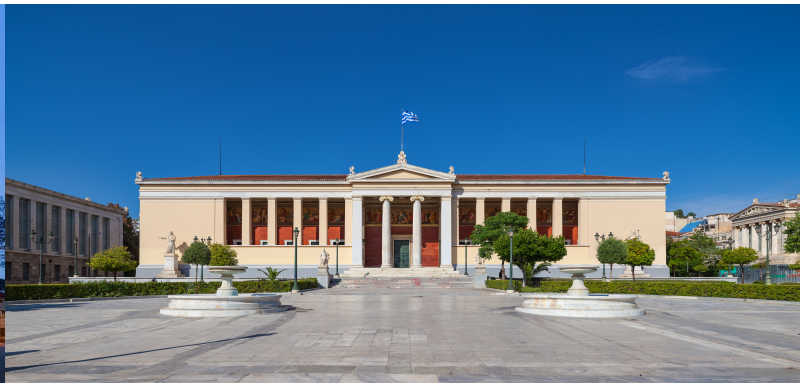

The 5th Symposium on Hypercompositional Algebra new Developments and Applications

Book of Abstracts

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UNIVERSITY OF ATHENS

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On the class of Transposition Hypergroups

Christos Massouros and Gerasimos Massouros
National and Kapodistrian University of Athens, Greece,
Hellenic Open University, Greece

Abstract

The introduction of a third axiom in hypergroups—the transposition axiom—has opened up a vast area of research within hypercompositional algebra. The class of transposition hypergroups includes structures such as quasicanonical hypergroups (or polygroups), canonical hypergroups, join spaces (or join hypergroups), fortified transposition hypergroups, fortified join hypergroups, and others. Some of these structures are used in constructing more complex hypercompositional systems—such as hyperfields, hyperrings, hypermodules, etc.—whose additive component is a canonical hypergroup, or equivalently, a commutative transposition hypergroup with scalar identity. Several open problems remain concerning the development and analysis of analogous structures in which the additive part is a different type of transposition hypergroup, e.g., a fortified join hypergroup.

HAnDA 2025

AI Tools and the Exploration of Hypercompositional Structures in Algebra

Charalampos Tsitouras
National and Kapodistrian University of Athens, Greece

Abstract

The recent proliferation of artificial intelligence tools, particularly large language models such as ChatGPT, offers powerful new methods for exploring algebraic systems defined by hypercompositions. Hypercompositional structures, such as hypergroups, hypergroupoids, hyperfields, hyperrings etc, extend classical algebraic notions by allowing operations to produce sets of outcomes rather than single values. These systems are foundational in generalized algebra and have applications in automata theory, fuzzy systems, and non-classical logics. This lecture presents how AI tools can assist in the construction, enumeration, classification, and analysis of hypercompositional structures. Using case studies—such as the enumeration of hypergroups of small orders, verification of structural properties like associativity and reproductivity, and isomorphism detection—we demonstrate how symbolic computation environments (e.g., Mathematica) combined with natural language interfaces (e.g., ChatGPT) can streamline both research and pedagogical tasks. In particular, we emphasize how various results—such as those of Massouros and Tsitouras on the enumeration of hypergroups—can be interactively verified and further explored with the support of AI tools. The presentation will conclude with a discussion on the prospects of integrating machine learning with algebraic hypercompositional structures, potentially automating the discovery of structural invariants and classifying large families of algebraic structures.

Can Hypercompositional Algebra contribute to the resolution of the unsolved problems of modern Physics?

Spyridon Vossos and Elias Vossos

National and Kapodistrian University of Athens, Greece

Abstract

Hypercompositional Algebra (HA), a generalization of classical algebraic systems through multi-valued operations, can offer a novel mathematical lens for addressing some of the unresolved challenges in modern physics. By extending traditional structures such as groups, rings, and fields into hypercompositional structures—where binary operations yield sets rather than single elements—HA introduces tools capable of modeling ambiguity, non-determinism, and complex systemic interactions. This presentation explores the potential of HA to inform theoretical developments in areas such as Newtonian physics, Special Relativity (SR), General Relativity (GR), Quantum mechanics, Quantum field theory, Symmetry breaking, and Quantum gravity. More specifically, we examine if the Linear Transformations:

- Closed Isometric Complex Boost in Isotropic Complex Spacetime and
- Open Isometric Generalized Real Boost in Isotropic Real Spacetime,

which led to Generalized Special Relativity (which Unifies the Newtonian Physics and Einsteinian Relativity Theory), could also lead to a new Hypergroup (which includes both the Galilean group and Lorentz group).

Moreover, we examine if the Scale factor a(t) in the FLRW metric (which is used in the Study of the expansion of Universe), could also be studied via the Hypercompositional Algebra.

HAnDA 2025

Supplementing Krasner Hypermodule

Burcu Nişancı Türkmen

Department of Mathematics, Faculty of Sciences and Arts, Amasya University, Turkey

Abstract

In this talk, it is characterized supplementing Krasner hypermodules in the subcategory ${}_{R_h}hmod$. It has been proven that the hypermodule, whose the subhypermodule and the factor hypermodules with its subhypermodule and factor hypermodule are supplementing hypermodules, is itself a supplementing Krasner hypermodule. The transfer of hyperrings in the category ${}_{R_h}hmod$ to factor hyperrings has been associated with supplementing hypermodules. It has been demonstrated that supplementing hypermodules with normal projective and reduced have small radicals.

Anti-Ideal-Theory in Algebraic Structures and their Fuzzifications

Al-Tahan Madeleine

Department of Mathematics and Statistics Abu Dhabi University

Abstract

Algebraic structures play a fundamental role in mathematics and have a wide range of applications. However, analyzing the properties and behavior of an entire algebraic structure can often be complex and challenging. To make this task more manageable and focused, researchers frequently study specific substructures or subsets such as ideals, filters, or anti-ideals.This talk explores algebraic structures through the lens of various types of anti-ideals. These subsets often involve an inherent degree of vagueness or uncertainty. To address this, we introduce their fuzzification, which provides a more flexible and realistic framework for describing such processes within algebraic systems.

HAnDA 2025

On Normal Projective (Injective) Krasner Hypermodules

Ergül Türkmen

Department of Mathematics, Faculty of Sciences and Arts, Amasya University, Turkey

Abstract

In this talk, with the help of splitting short exact sequences, characterizations of normal projective and normal injective hypermodules will be provided. In particular, a characterization of hyperrings whose all hypermodules are normally injective (projective) will be given.

HAnDA 2025

Fuzzy Graph Hyperoperations

Antonios Kalampakas

American University of the Middle East

Abstract

We introduce novel hypercompositional structures on fuzzy graphs by extending classical graph hyperoperations into the fuzzy domain. Building on the concept of strongest strong paths—those that are both optimal in strength and composed entirely of strong edges—we define two types of fuzzy hyperoperations: a vertex-based and an edge-based variant. These operations generalize established crisp hyperoperations and retain compatibility with the underlying graph topology.

HAnDA 2025

Construction of finite fields and hyperfields

Bianca-Liana Bercea

Ovidius University from Constanta

Abstract

We can ask ourselves how we can construct new examples of finite fields. Analyzing the field of complex numbers and the fields $\mathbb{Z}_p$ with the prime number p, we will notice that we can construct other examples of fields in the following way: we take a commutative ring A in which Euclid's algorithm can be applied, consider p a prime element, and take the set of residue classes upon division by p, denoted A_p. Then A_p will have the canonical field structure. Using these finite fields, we can construct certain finite hyperfields.

HAnDA 2025

Some classes of hypernear-rings

Sanja Jančić-Rašović, Anton Nuculović

University of Montenegro

Abstract

In this presentation, we are going to talk about three types of hypernear-rings and their main characteristics. First, we will look at hypernear-rings and their elements that "correct" the lack of distributivity, in other words we deal with the elements that assume the validity of distributivity. Next, we will cover division hypernear-rings, which form a special class with unique features. Lastly, we will present findings on general hypernear-rings that are associated with a hypergroup.

HAnDA 2025

Analysis of Weak Associativity in Some Algebraic Hypercompositional Structures that Represent Dismutation Reactions

Temitope Gbolahan JAIYEOLA, Kehinde Gabriel ILORI and Oyeyemi Oluwaseyi OYEBOLA

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Abstract

In this paper, some chemical systems of Tin (Sn), Indium (In) and Vanadium (V) which are represented by algebraic hypercompositional structures $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ were studied. The analyses of their algebraic properties and the probabilities of elements in dismutation reactions were carried out with the aid of computer codes in Python programming language. It was shown that in the dismutation reactions, the left nuclear (N_λ)-probability, middle nuclear (N_μ)-probability and right nuclear (N_ρ)-probability for each of the algebraic hypercompositional structures $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ is less than 1.000. This implies that, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ are non-associative algebraic hypercompositional structures . Also, from the results obtained for FLEX-probability, it was shown that, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ have flexible elements because the values of their FLEX-probabilities are 1.000 each. Hence, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ are flexible. Overall, $(S_V, \oplus)$ exhibited the lowest measure of weak-associativity, $(S_{Sn}, \oplus)$ exhibited lower measure of weak-associativity, and $(S_{In}, \oplus)$ exhibited a low measure of weak-associativity.

HAnDA 2025

Analysis of Weak Associativity in Some Algebraic Hypercompositional Structures that Represent Dismutation Reactions

Oyeyemi Oluwaseyi Oyebola

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Abstract

This study presents a characterization of the extra polyloop-3, i.e., a particular class of polyloops distinguished by the satisfaction of the third extra identity: $xy \cdot xz = x(yx \cdot z)$, in the classical non-associative algebraic structure, the one of extra loop. This gives an equivalence of seven identities in the corresponding algebraic non-associative hypercompositional structure, called extra polyloop-3. In contrast to the classical non-associative algebraic structures, the extra loops, where the binary operation satisfies any of the three extra identities, non-associative algebraic hypercompositional structures exhibit more distinctly dissimilar and complex behaviours. We study the interactions between the nonassociativity of extra polyloop-3 and other algebraic properties such as the flexibility law, left alternative property, right alternative property, and power associativity.

HAnDA 2025

Fuzzy aspects in biological inheritance

Andromeda Pătrașcu Sonea

Ion Ionescu de la Brad Iasi University of Life Sciences

Abstract

Genetics is the science that studies the heredity and variability of organisms. Genetics explains the mechanisms of recording, modifying, and transmitting hereditary information from generation to generation. Biologist and mathematician Gregor Mendel is considered the founder and father of genetics. He developed the theory of hereditary factors, according to which a specific material particle determines each character of the organism, called a hereditary factor (gene). The study of hyperstructures in this context is necessary because the results of the experiments in the case of the monohybrid cross in the pea plant, the dihybrid cross in the pea plant, the inheritance in the Four-o' clock plant, and so on describe hypergroups. This paper will emphasize the fuzzy function associated with the classes formed by the phenotypes resulting from the simple dominance (dihybrid cross, trihybrid cross, 4-hybrid cross). We have chosen to study this because it has been observed that there is a link between the fuzzy function and the number of resulting phenotypes.

HAnDA 2025

Encoding hypercompositional structures through binary strings

Enrico Talotti

University of Nova Gorica

Abstract

We propose a novel computational framework for implementing and analyzing hypercompositional structures using Boolean algebras and associated isomorphisms. This approach enables the representation of the hypercomposition table as integer valued matrices, facilitating their manipulation and structural analysis using native binary operations, such as AND, OR, and bit shifts, available in all modern programming languages. Our goal is to create a library written in Rust that enables the generation of algebraic hypercompositional structure, as well as the identification of some specific structure like (semi)hypergroups, transposition hypergroups, quasi-caonical and canonical hypergroups etc, and the investigation of the corresponding substructures. We also implement the identification of specific elements such as scalar elements, identities, scalar identities, etc.

HAnDA 2025

On the Generation of Finite Real Hyperfields with Cyclic Positive Cones

Hanna Stojałowska, Katarzyna Kuhlmann and Dawid Kędzierski

Department of Mathematics West Pomeranian University of Technology Szczecin, Poland

Abstract

During the talk we will focus on finite real hyperfields. In particular, we pay special attention to finite real hyperfields with cyclic positive cones. We present an algorithm for determining all such hyperfields up to isomorphism and compute their C-characteristic. We will also present the results of a computer program that provides examples and data on the number of such hyperfields of order up to 13, as well as the C-characteristics occurring in such hyperfields of order up to 17.

HAnDA 2025

On the notion of algebraically closed hyperfields

Alessandro Linzi

University of Nova Gorica

Abstract

Algebraically closed hyperfields are defined in the literature, by analogy with field theory, as hyperfields where all polynomials have a root. We consider an example of an algebraically closed hyperfield motivating a broader perspective on the definition of algebraically closed hyperfields. We speculate on the consequences of this broader perspective connecting it to the notion of existentially closed structures.

HAnDA 2025

Support of a Hypermodule

Hashem Bordbar

University of Nova Gorica, Slovenia

Abstract

We initiate the study of the notion of *support* for hypermodules over Krasner hyperrings, establishing several intrinsic connections with the concept of annihilators in this setting. After defining and characterizing the support of a hypermodule, particularly for finitely generated hypermodules, we employ the notion of hypermodule length to further explore its structure. As a main result, we determine the support of quotient hypermodules that contain a unique maximal hyperideal.

On Morgado and Sette's Implicative hyperlattices as models of da Costa logic C_ω

Marcelo E. Coniglio

University of Campinas, Brazil

Abstract

José Morgado introduced in 1962 an original and interesting notion of hyperlattices, that he called *reticuloides*. In his Master's thesis defended in 1971 (and supervised by Newton da Costa), Antonio M. Sette proposed a novel notion of implicative hyperlattices (here called SIHLs) based on Morgado's hyperlattices. He also extended SIHLs by adding an unary hyperoperator, obtaining a class of hyperalgebras (here called SHCωs) which correspond to da Costa's algebras for $C\omega$, being so a suitable semantics for the well-known da Costa's logic C_ω. In this talk we show new interesting results about Morgado's hyperlattices and Sette's implicative hyperlattices. In particular, a natural characterization of SIHLs in purely (hyper)lattice-theoretic terms will be proved, showing that, as in the algebraic case, they are distributive. We also introduce a class of swap structures, a special class of hyperalgebras over the signature of C_ω naturally induced by implicative lattices. It is proven that these swap structures are indeed SHCωs. Finally, it is proven that the class of SHCωs, as well as the above mentioned class of swap structures, characterize the logic C_ω. We argue that Morgado's hyperlattices constitute a very natural notion of hyperlattices, generalizing the algebraic case (based on partial orders) to hyperalgebras, by moving to preorders. This is a joint work with Ana C. Golzio and Kaique M. Roberto.

HAnDA 2025

Hyper swap structures

Kaique M. Roberto

Faculdade Einstein, São Paulo, Brazil

Abstract

Non-deterministic matrices (a.k.a. Nmatrices) constitute a natural generalization of logical matrices, in which an algebra of truth-values is replaced by an hyperalgebra. Valuations over Nmatrices "pick", for every formula, a single element of the set of values produced by the hyperoperator associated to the main connective of the formula, when applied to the value assigned to its immediate subformulas. This notion was formalized in 2001 by Avron and Lev, but it was already used by several authors, notoriously by Ivlev in the 1970s in the context of non-normal modal logics. In some cases, finite-valued Nmatrices provide decision procedures for logics that cannot be characterized by a single finite-valued matrix. In 2016 Carnielli and Coniglio introduced the notion of swap structures, which are hyperalgebras

formed by elements of a given algebra (typically a Boolean algebra or a Heyting algebra) representing semantical states of a formula in terms of formulas of the language of the underlying algebra. The hyperoperators are defined by imposing relational conditions to (some of) the coordinates of the input states. Swap structures can be seen as non-deterministic twist structures. In this talk we introduce the notion of hyper swap structures, which are swap structures based on an hyperalgebra. This natural generalization of swap structures allows to construct equivalences between categories of hyperalgebras, as happens with twist structures in the deterministic (algebraic) case. As an illustrative example, it will be shown that the categories of hyperalgebras for da Costa's logic C_ω and the category of Sette's implicative hyperlattices are equivalent. This is a joint work with Marcelo E. Coniglio and Ana C. Golzio.

HAnDA 2025

Weakly free multialgebras

Guilherme Toledo

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Center for Logic, Epistemology and History of Science, University of Campinas, Brazil

Abstract

In abstract algebraic logic, many systems, such as those paraconsistent logics taking inspiration from da Costa's hierarchy, are not algebraizable by even the broadest standard methodologies, as that of Blok and Pigozzi. However, these logics can be semantically characterized by means of non-deterministic algebraic structures such as Nmatrices, RNmatrices and swap structures. These structures are based on multialgebras, which generalize algebras by allowing the result of an operation to assume a non-empty set of values. This leads to an interest in exploring the foundations of multialgebras applied to the study of logic systems.

It is well known from universal algebra that, for every signature, there exist algebras over which are absolutely free, meaning that they do not satisfy any identities or, alternatively, satisfy the universal mapping property for the class of algebras. Furthermore, once we fix a cardinality of the generating set, they are, up to isomorphisms, unique, and equal to algebras of terms (or propositional formulas, in the context of logic). Equivalently, the forgetful functor, from the category of algebras to Set, has a left adjoint. This result does not extend to multialgebras. Not only multialgebras satisfying the universal mapping property do not exist, but the forgetful functor, from the category of multialgebras to Set, does not have a left adjoint.

In this paper we generalize, in a natural way, algebras of terms to multialgebras of terms, whose family of submultialgebras enjoys many properties of the former. One example is that, to every pair consisting of a function, from a submultialgebra of a multialgebra of terms to another multialgebra, and a collection of choices (which selects how a homomorphism approaches indeterminacies), there corresponds a unique homomorphism, what resembles the universal mapping property. Another example is that the multialgebras of terms are generated by a set that may be viewed as a strong basis, which we call the ground of the multialgebra. Submultialgebras of multialgebras of terms are what we call weakly free multialgebras. Finally, with these definitions at hand, we offer a simple proof that multialgebras with the universal mapping property for the class of all multialgebras do not exist and that does not have a left adjoint.

The Fuzzy Grade of a certain class of Hypergroups

Violeta Fotea

Al. I. Cuza University, Iasi, Romania

Abstract

In this presentation, we investigate the fuzzy degree of a hypergroup, which can be associated with genetic heredity. We analyze different cases for the cardinality of the initial hypergroup. This study is a continuation of a previous study.

HAnDA 2025

Arithmetic functions on complete hypergroups

Irina Cristea

University of Nova Gorica, Slovenia

Abstract

In this presentation, I will explore several arithmetic functions defined on complete hypergroups. These functions possess a rich combinatorial significance and establish connections with graph theory, fuzzy set theory, as well as various topics in engineering and information technology. Specifically, I will discuss the fuzzy and intuitionistic fuzzy grade of a complete hypergroup, a generalization of Euler's totient function, and the commutativity degree of a complete hypergroup. Finally, I will examine how these concepts relate to dependence relations, which will be the focus of the concluding part of the presentation.